Proceedings of the 2006 Crystal Ball User Conference

STOCHASTIC CRITICAL PATH

Eduardo Herrera Lana
CYDHEM S.A.
Toledo N23-126 y Madrid
Quito, Ecuador

ABSTRACT

Determining the critical path has been the right way for managing schedules in projects. Most of the time using this approach does not guarantee that the project will finish on time. Some project managers use the most likely scenario for each task in the project, and other use three scenarios: pessimistic, optimistic, and most likely. Each method is problematic: the project will not reach the goals on time because the project manager does not know anything about the probability of each scenario. Simulation is the best way to determine all scenarios in which it is possible to trace the stochastic critical paths. With simulation, project managers can guarantee more realistic time forecasts.

1 INTRODUCTION

The Critical Path Method (CPM) was invented by DuPont in 1957 for providing directions on how to control the schedule of some specific projects. From that time, there have been many different applications in all types of industries. The goal of the method is to estimate the total duration of a project by determining the End Last Finish Time (ELFT). This is done by considering just the activities that do not have slack, because a delay on them it implies a delay on the total time required for finishing the project on time. The activities without slack trace the critical path of the project schedule.

CPM requires project managers to define the relevant activities, including estimates of their durations, and the sequence of the activities. The output is usually represented by a network diagram in which the critical path is identified. The main weakness of the method is that, when considering certain time forecasts for each activity in the project, to the scheduler determine just one critical path that most of the time underestimates the total completion time.

2 DISCUSSION

Suppose a project of fourteen relevant activities, described in Table 1. In order to identify the critical path of the project, it is necessary to determine, for each activity:

1. The Earliest Starting Time (EST),
2. The Earliest Finishing Time (EFT),
3. The Latest Finishing Time (LFT),
4. The Latest Starting Time (LST), and
5. The Slack

The earliest times are determined by working forward through the diagram or table to include all the activities, as opposed to the latest times that are calculated by working backward. The slack is the difference between either the earliest times or the latest times. Applying this approach to the example, the End Last Finish Time (ELFT) for the project is 290 days, and the activities that lie on a critical path are: A, C, E, F, J, M and N. That means if a delay occurs in any of these activities, the total completion time will be over 270 days.

Scenario analysis determines the best-case, most likely, and worst-case scenarios regarding to the duration of each relevant activity on a project. Applying CPM to each scenario allows a project manager to identify a range for the completion time but not the associated probabilities of occurrence. Table 2 shows the scenarios for the example above. The completion time range is 198 to 395 days.

Table 1: Single Point Project Scheduling

Activities		**Sequence**						**Duration**
Id	Description	Predecessor			Subsequent			LTD
	Start							
A	Task A				C			40
B	Task B				D			20
C	Task C	A			E			25
D	Task D	B			E			25
E	Task E	C	D		F	G	H	45
F	Task F	E			I	J		60
G	Task G	E			K			45
H	Task H	E			L			45
I	Task I	F			M			35
J	Task J	F			M			40
K	Task K	G			M			30
L	Task L	H			N			32
M	Task M	I	J	K	N			20
N	Task N	L	M					40

Table 2: Scenario Analysis for Project Scheduling

Activities		**Durations**		
Id	Description	OTD	LTD	PTD
	Start			
A	Task A	30	40	60
B	Task B	15	20	30
C	Task C	15	25	40
D	Task D	20	25	40
E	Task E	40	45	65
F	Task F	30	60	95
G	Task G	35	45	60
H	Task H	40	45	60
I	Task I	30	35	50
J	Task J	30	40	60
K	Task K	25	30	60
L	Task L	28	32	42
M	Task M	18	20	25
N	Task N	35	40	50

It is easy to think of adopting “the average” or “the most important value” for a deterministic analysis because of the tendency to overestimate their meaning. In fact, the problem is the sequence observed in many natural ways. That sequence creates another dimension of analysis because “the average” or “the most important value” of one separate event is not independent and has meaning in context to other events. For example, for five sequential events with each having a 50% probability of occurrence, the probability of the sequence is = power(0.5,5) = 3.1%,. That means that there is 97% confidence that the five sequential events will not occur. This is a critical situation for managing schedule in projects, because of the sequence required for completion.

3 SIMULATION MODEL

The only way to capture the uncertainty in the estimation of the total completion time is by creating a dynamic model that simulates not a few scenarios, but all possible scenarios. The static model in Microsoft® Excel must be transformed into a simulation model using Crystal Ball®. The parameters for defining the model are:

- Assumptions: the duration of each activity is represented by a Gamma distribution (e.g., Figure 1)
- Forecast: the total completion time

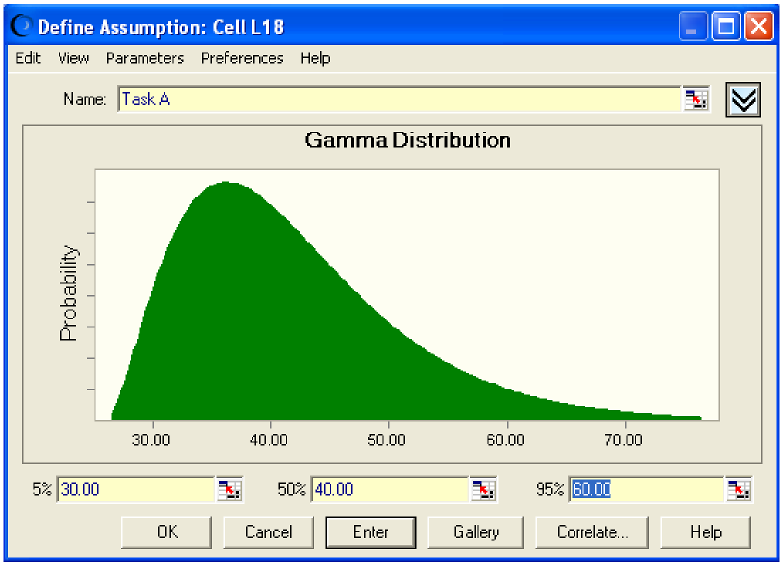

Figure 1: Stochastic duration of an activity

A skewed distribution represents the duration of an activity better than a triangular distribution, because it includes the upside values. In general, the main difficulty for using the gamma distribution is in how to estimate its parameters. Crystal Ball allows users to define alternate parameters, for instance, the percentiles P5, P50, and P95.

A Path Control Box (PCB) has been created into the model to show us how the critical path is not only one critical path (see Figure 2). In fact, there are many potential critical paths depending on the conditions surrounding the time variables necessary to conclude the project.

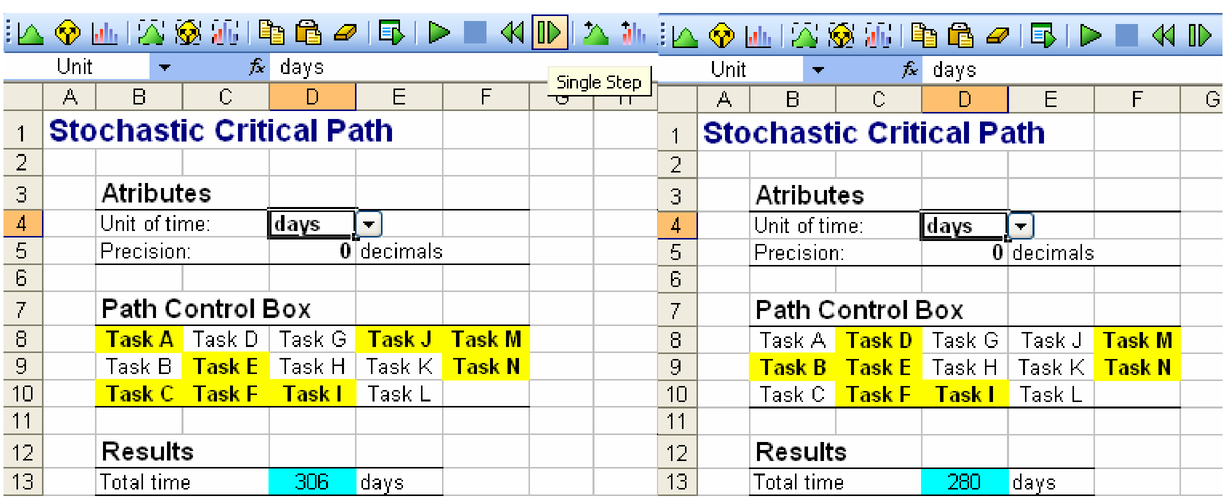

Figure 2: Stochastic Critical Paths

4 RESULTS

Figure 3 shows the forecast chart for the total completion time for the project, after 10,000 trials. The probability of not exceeding the base case time is approximately 30%. The inputs whose variability contribute the most for the dispersion in the total completion time are Task 1, Task 2, and Task 3. They contribute over the 75% to the variance on the forecast (Figure 4).

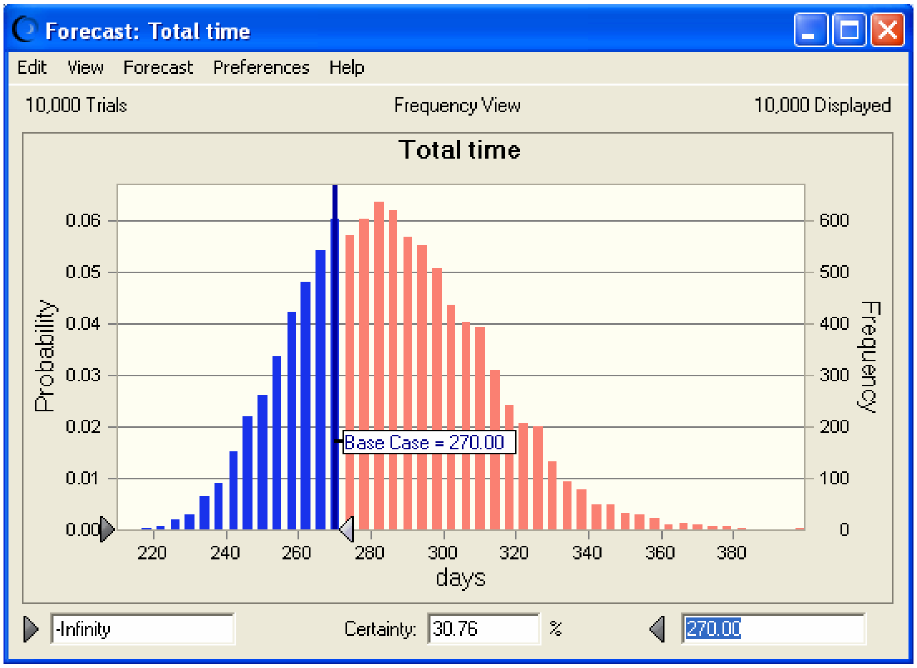

Figure 3: Total completion time forecast

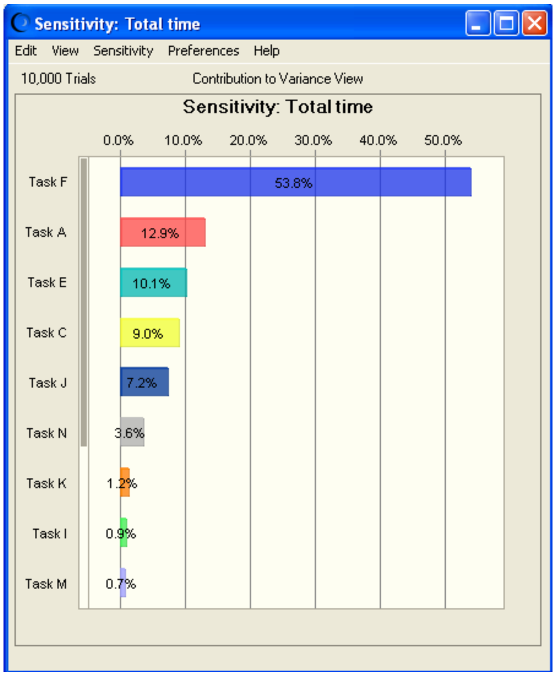

Figure 4: Sensitivity of the total completion time

Figure 5 shows the number of activities lying on a critical path for the project example, and Figure 6 displays the sensitivity of the forecast.

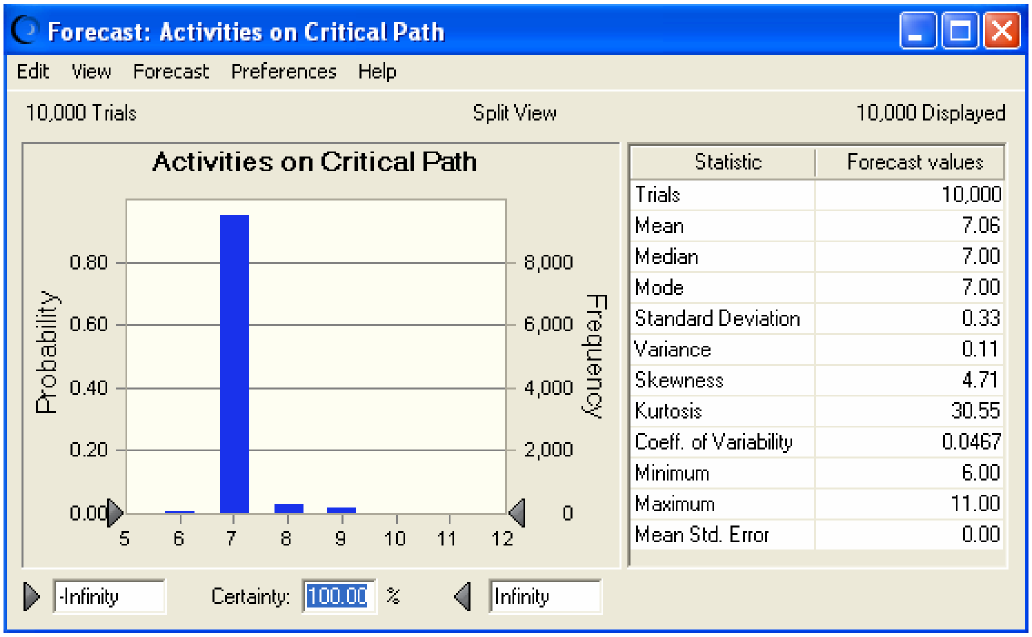

Figure 5: Number of activities on a critical path

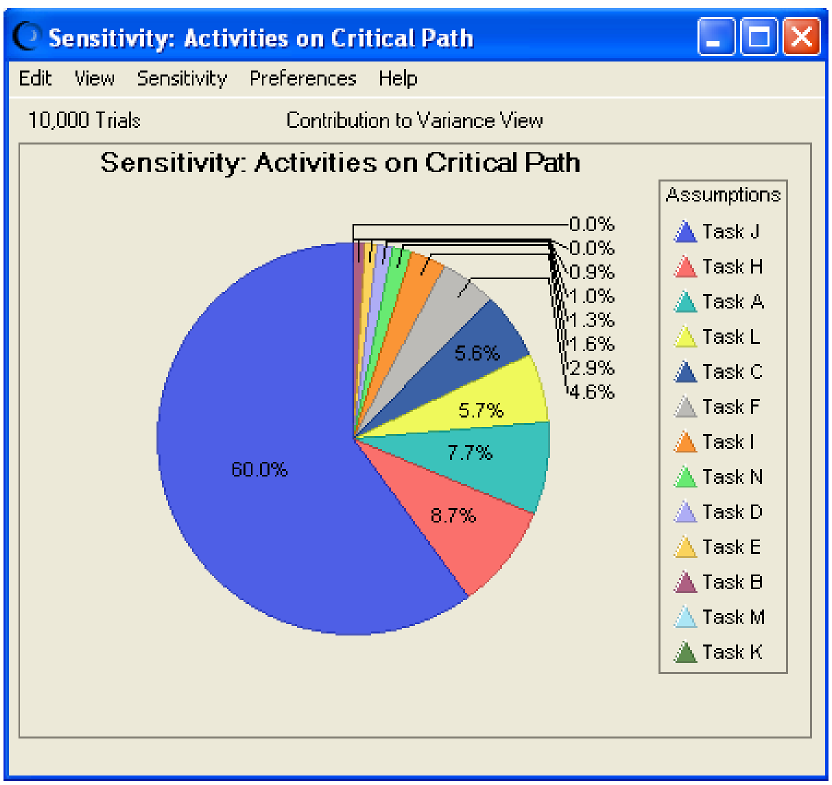

Figure 6: Sensitivity of the number of activities on a critical path

5 CONCLUSION

The goal of estimating stochastic critical paths in project management is to optimize the forecasts for improving the use of the most important future resource: time. Estimating the total completion time using Monte Carlo simulation is the best way to manage in project schedules because it allows project managers to associate probabilities with the results. In addition, this approach shows us that more than one critical path is possible, depending on the uncertainty of the assumptions and the goodness of the activity sequence. The activities whose uncertainties have the greatest influence on the total completion time are not necessarily the same activities defining the most critical paths. That means, one particular critical path is important depending on how probable it is. For this reason, using a single-point estimate or scenario analysis could be very inappropriate.

REFERENCES

Decisioneering, Inc. Crystal Ball User Manual. *USA, 2005*.

BIOGRAPHY

Eduardo Herrera Lana (eduardo.herrera@cidem.com.ec) is the Chief Executive Officer of CYDHEM S.A., a company providing business solutions in Latin America. Eduardo has applied Monte Carlo simulation since 1996. For over a decade, Eduardo has trained to a wide variety of executives and postgraduates in fields such as Project Management, Finance, Simulation Techniques, and Quantitative Methods for Making Decisions. Eduardo has been certified by Decisioneering, Inc. to teach Crystal Ball Software. He has a MBA (Summa Cum Laude) from the National Polytechnic School (Quito, Ecuador) and a Pedagogy diploma from the Army Polytechnic School (Quito, Ecuador). He has also assisted many companies to modeling real life problems.